

Izhikevich Model of Cortical Spiking Neurons

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Introduction

Neurons are the elementary processing units in the brain and central nervous system. When a neuron receives an appropriate stimulus, it produces *action potentials* (electrical impulses) that are propagated along its axon. When the pulse reaches the end of the neuron, other neurons or muscle cells may be activated. There are three types of neurons:

1. Sensory Neurons: receive stimuli from sensory organs
2. Inter-neurons: transfer information from one neuron to another
3. Motor Neurons: transfer information about the control of muscle cells

The sending neuron is called the *pre-synaptic cell* and the receiving neuron is called the *post-synaptic cell*. A neuron consists of three functionally distinct parts:

1. Dendrites: (input) collects signals from other neurons and transmits them to the soma
2. Soma: (processing) if the total non-linear input signal from the dendrites is greater than some threshold, an output signal is generated
3. Axon: (output) electric signals are propagated away from the soma to other neurons across junctions called *synapses*

The inside of the axon is filled with an ionic fluid that is separated from the surrounding body fluid by a thin membrane. The ionic solutes in the extracellular fluid are primarily Na^+ and Cl^- ions. The intracellular fluid is dominated by K^+ and large organic anions. In the resting state, the axon membrane is highly permeable to K^+ ions, slightly permeable to Na^+ ions, and impermeable to large organic anions. Since more K^+ ions leak out of the cell than Na^+ ions that leak into the cell, the inside of the cell is more negative than the outside. This potential difference is called the *membrane potential* $v_m(t)$. The outside of the cell is taken as the reference potential 0 V. The resting membrane potential is often -65 mV.

Simulation of Action Potential

The mechanism for the generation of an electrical signal by a neuron is conceptually simple. When a neuron receives a sufficient stimulus from other neuron, the permeability of the cell membrane changes. As a result, there is an influx of Na^+ ions into the cell while K^+ ions leave. The movement of ions across the membrane constitute an electric current signal, which propagates along the axon to its terminals. These membrane currents depolarize the cell and generates a voltage signal, which are called *spikes*. The duration of each spike is less than a few milliseconds or less and has a peak voltage of about +100 mV.

We can simulate an action potential using the *Hodgkin-Huxley model*. Hodgkin and Huxley developed a mathematical model to explain the behavior of nerve cells in a squid giant axon in 1952. Their model was developed well before the advent of electron microscopes or computer simulations, and gave scientists a basic understanding of how nerve cells work without having a detailed understanding of neurobiology. This will be done later in the course.

Introduction to Izhikevich Model

Most neurons are excitable in that they can fire a voltage spike when stimulated. In this way, a computer simulation of a neuron network needs to account for and explain the patterns producing by biological cells. To do this, we must satisfy two requirements:

1. The model must be computationally simple.
2. The model must be able to produce the rich firing patterns exhibited by real biological neurons.

The Hodgkin-Huxley model previously described is computationally prohibitive, and can only simulate a few neurons in real-time. Our goal is not to simulate the behavior of a population of neurons in real-time.

We will explore the model proposed by *Eugene Izhikevich*. Unlike the Hodgkin-Huxley model, the Izhikevich model does not account for the biophysics of neurons. It uses mathematical equations to compute a wide range of spiking patterns for cortical neurons. The output is incredibly realistic and biologically plausible.

The value of the four parameters ***a***, ***b***, ***c***, and ***d*** used in the Izhikevich model determines the spiking and bursting behavior of the known types of cortical neurons. The time evolution of the membrane potential ***v*** is described by the differential equations:

1. $\frac{dv}{dt} = c_1 v^2 + c_2 v + c_3 - c_4 u + c_5 I$
2. $\frac{du}{dt} = a (bv - u)$

The after-spike setting relationship is:

1. If $v \geq +30mV$, then $v \rightarrow c$ and $u \rightarrow u + d$ where u is the membrane recovery variable.

The dimensions and values of the model parameters are:

| | |
|-----------------|--|
| v | membrane potential [mV] |
| t | time [ms] |
| $\frac{dv}{dt}$ | time rate of change in membrane potential [mV/ms] |
| u | recovery variable [mV] |
| I | external current input to cell (synaptic currents or injected DC-currents) |

The constants are:

1. $c_1 = 0.04 \frac{mV}{ms}$
2. $c_2 = 0.05 \frac{1}{ms}$
3. $c_3 = 140 \frac{mV}{ms}$
4. $c_4 = 1 \frac{1}{ms}$
5. $c_5 = 1 \frac{mV \cdot \Omega}{ms \cdot A \cdot ms}$

The membrane recovery variable u provides negative feedback to the membrane potential v and accounts for the activation of the K^+ currents and inactivation of the Na^+ currents. The term $c_1 + c_2 v + c_3$ was obtained by fitting the spike initiation dynamics of a cortical neuron so that the membrane potential v has units of mV and the time t is in ms. The model does not have a fixed threshold, which most real neurons have. Instead, the firing depends on the history of the membrane potential prior to a spike. The threshold potential may be in the range of -55 mV to -40 mV. The resting potential in the model is between -70 mV and -60 mV depending on the value of the parameter ***b***.

/Users/naureenghani/Downloads/fig1.png

Simulation of Izhikevich Model

Various choices of the parameters *a*, *b*, *c*, and *d* result in the various intrinsic firing patterns than can be computed:

- $a \sim 0.02 \frac{1}{ms}$ determines the time scale of the recovery variable *u*. The larger the value of *a*, the quicker the recovery.
- $b \sim 0.20$ [dimensionless] describes the sensitivity of the recovery variable *u* to the sub-threshold fluctuations of the membrane potential *v*. Larger values of *b* couple *u* and *v* more strongly, resulting in possible sub-threshold oscillations and low-threshold spiking dynamics.
- $c \sim -65mV$ gives the after-spike reset value of the membrane potential *v* caused by the fast high-threshold K^+ conductances.
- $d \sim 6mV$ describes the after-spike reset of the recovery variable *u* caused by slow high-threshold Na^+ and K^+ conductances.

Objective

A neuron can be stimulated by the injection of DC current pulse through an electrode. We record the response of the membrane potential. When a step-input current is used to stimulate a neuron, the cell continues to fire a sequence of spikes called a *spike train*. From such investigations, scientists uncovered an array of firing patterns in the cortex. The objective of the Izhikevich model is to reproduce these patterns using computation.

Code

```
% Simulation of Cortical Spiking Neurons
% Source:
% E M Izhikevich: Simple Models of Spiking Neurons (2004)

% Step 1: Clean workspace
close all; % close open figures
clc; % clear command window
clear % clear workspace

% Step 2: Input parameters
flagS = 1; % select 1 of 20 spike train patterns
N = 5000; % number of grid points
tMax = 100; % max time [ms]

% Step 3: Define model parameters [a b c d I] 1x5 matrix for each one of 20
% patterns
% a, b, c, and d simulate the neuron
% I simulates the current injection
% Note:
% current injection = voltage response
% voltage injection = current response

coeffM = zeros(20,5); % pre-allocate 20x5 matrix of zeros
coeffM(1,:) = [0.02 0.4 -65 2 10]; % tonic spiking
coeffM(2,:) = [0.02 0.25 -65 6 1]; % phasic spiking
coeffM(3,:) = [0.02 0.20 -50 2 15]; % tonic bursting
coeffM(4,:) = [0.02 0.25 -55 0.05 0.6]; % phasic bursting
coeffM(5,:) = [0.02 0.20 -55 4 10]; % mixed mode
coeffM(6,:) = [0.01 0.20 -65 8 30]; % spike frequency adaptation
coeffM(7,:) = [0.02 -0.1 -55 6 0]; % Class I
coeffM(8,:) = [0.20 0.26 -65 0 0]; % Class II
coeffM(9,:) = [0.02 0.20 -65 6 7]; % spike latency
coeffM(10,:) = [0.05 0.26 -60 0 0]; % sub-threshold oscillations
```

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coeffM(11,:) = [0.10 0.26 -60 -1 0]; % resonator
coeffM(12,:) = [0.02 -0.1 -55 6 0]; % integrator
coeffM(13,:) = [0.03 0.25 -60 4 0]; % rebound spike
coeffM(14,:) = [0.03 0.25 -52 0 0]; % rebound burst
coeffM(15,:) = [0.03 0.25 -60 4 0]; % threshold variability
coeffM(16,:) = [1 1.5 -60 0 -65]; % bistability
coeffM(17,:) = [1 0.2 -60 -21 0]; % DAP
coeffM(18,:) = [0.02 1 -55 4 0]; % accomodation
coeffM(19,:) = [-0.02 -1 -60 8 80]; % inhibition-induced spiking
coeffM(20,:) = [-0.026 -1 -45 0 80]; % inhibition-induced bursting

% Step 4: Use input parameters to simulate one of twenty patterns
a = coeffM(flagS,1);
b = coeffM(flagS,2);
c = coeffM(flagS,3);
d = coeffM(flagS,4);
I = coeffM(flagS,5);

% Step 5: Define all plot titles (20)
tm = [ ' 1: Tonic Spiking           '; ...
       ' 2: Phasic Spiking         '; ...
       ' 3: Tonic Bursting          '; ...
       ' 4: Phasic Bursting         '; ...
       ' 5: Mixed Mode              '; ...
       ' 6: Spike Frequency Adaptation'; ...
       ' 7: Class 1                 '; ...
       ' 8: Class 2                 '; ...
       ' 9: Spike Latency           '; ...
       '10: Subthreshold Oscillations'; ...
       '11: Resonator               '; ...
       '12: Integrator              '; ...
       '13: Rebound Spike           '; ...
       '14: Rebound Burst           '; ...
       '15: Threshold Variability    '; ...
       '16: Bistability             '; ...
       '17: DAP                     '; ...
       '18: Accomodation            '; ...
       '19: Inhibition-induced Spiking'; ...
       '20: Inhibition-induced Bursting'; ];

% Step 6: Solve Equations

% define c1, c2, c3:
cc(1) = 0.04; % [mv]/[ms]
cc(2) = 5; % [1]/[ms]
cc(3) = 140; % [mV]/[ms]

% pre-allocation
t = linspace(0,tMax, N); % time steps [ms]
v = zeros(N,1); vM = zeros(N,1); % membrane potential [mV]
u = zeros(N,1); uM = zeros(N,1); % recovery variable [mV]
Iext = zeros(N,1);

Iext(N/10:end) = I; % define size of I (initialization)
dt = t(2)-t(1); % one time step

% initialize values
v(1) = -65;

```

```

vM(1) = -65;
u(1) = b * v(1);
uM(1) = u(1);

% begin loop

for n = 1:N-1
    v(n+1) = v(n) + dt*( cc(1)*v(n)^2 + cc(2)*v(n) + cc(3) -u(n) + Iext(n) );
    u(n+1) = u(n) + dt*a*( b*v(n) - u(n));

    if v(n+1) > 30
        vM(n+1) = 30;
        v(n+1) = c;
        u(n+1) = u(n+1) + d;
    else
        vM(n+1) = v(n+1);
        uM(n+1) = u(n+1);
    end
end

% plot figure
figure(2); % 2 plots in one figure
set(gcf, 'units', 'normalized', 'Position', [0.3 0.32 0.32 0.38]);
%(gcf is current plot open (grab current figure)
% [x1 x2 y1 y2] provides range of x- and y- values to zoom in on
set(gca, 'fontsize', 12);
% gca is current plot open (grab current axes)
hSubplot = subplot(2,1,1);
% subplot is used to put >1 plot in one figure that is 2x1
% h = handle in hSubplot (important in graphical user interfaces (GUI) )
set(hSubplot, 'position', [0.13 0.46 0.78 0.45]);
xP = t; % x-axis is time
yP = vM; % y-axis is voltage
plot(xP, yP, 'b', 'Linewidth', 2);
ylabel('v_M [mV]');
title(tm(flagS,:)); % choose title from matrix of 20 entries

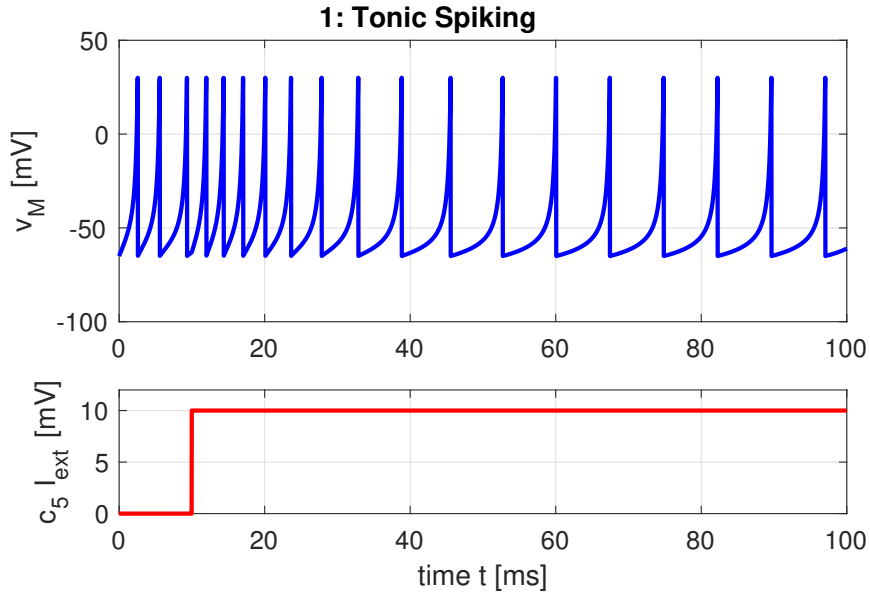
% plot voltage change based on current injection
grid on
set(gca, 'fontsize', 12);
hSubplot = subplot(2,1,2);
set(hSubplot, 'position', [0.13 0.15 0.78 0.20]);
xP = t;
yP = Iext; % external injection
plot(xP, yP, 'r', 'Linewidth', 2);
xlabel('time t [ms]');
ylabel('c_5 I_{ext} [mV]');
axis([0 tMax 0 1.2*max(Iext)]);
grid on
set(gca, 'fontsize', 12);

```

Tonic Spiking

$$a = 0.02 \quad b = 0.20 \quad c = -65 \quad d = 6$$

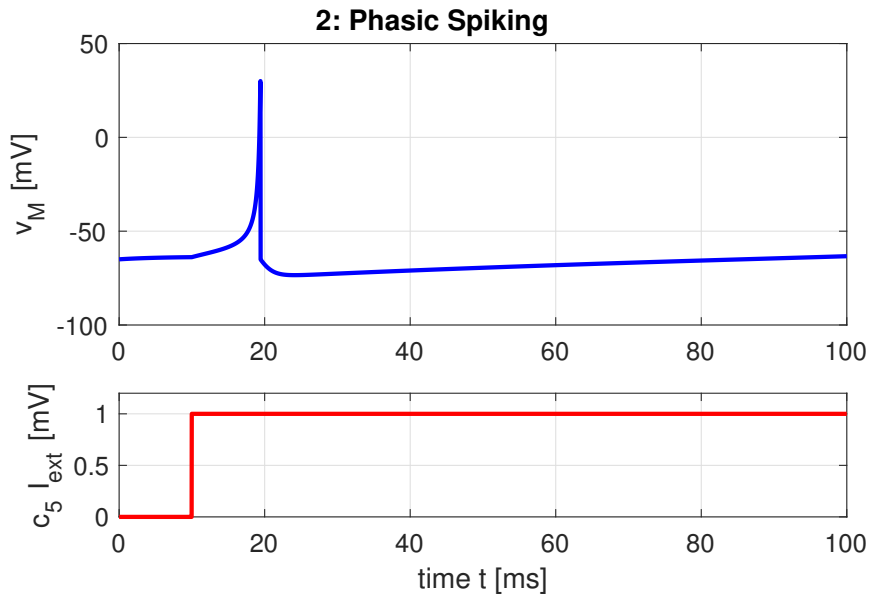
The most common type of excitatory neurons in the mammalian neocortex are pyramidal cells that fire spikes with decreasing frequency. When presented with a prolonged stimulus, the neurons fire a few spikes with a short inter-spike period and then the period increases. This is called *spike frequency adaptation* in biology. It means that the frequency is relatively high at the onset of the stimulation and then it adapts.



Phasic Spiking

$$a = 0.02 \quad b = 0.25 \quad c = -65 \quad d = 6$$

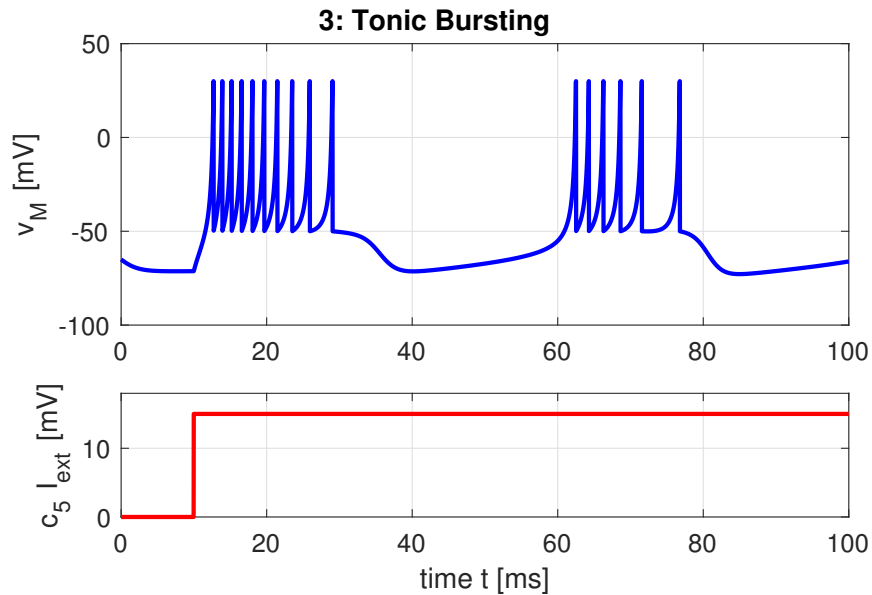
A neuron may only fire a single spike at the onset of the input stimulus and remain quiescent afterwards.



Tonic Bursting (excitatory “chattering” neurons)

$$a = 0.02 \quad b = 0.20 \quad c = -50 \quad d = 2$$

Some excitatory neurons such as *chattering neurons* in a cat cortex fire periodic bursts of closely spaced spikes when stimulated and may contribute to the gamma frequency oscillations in the brain. The inter-burst frequency can be as high as 50 Hz. The c parameter represents a very high reset voltage while d is a moderate after-spike jump of u .

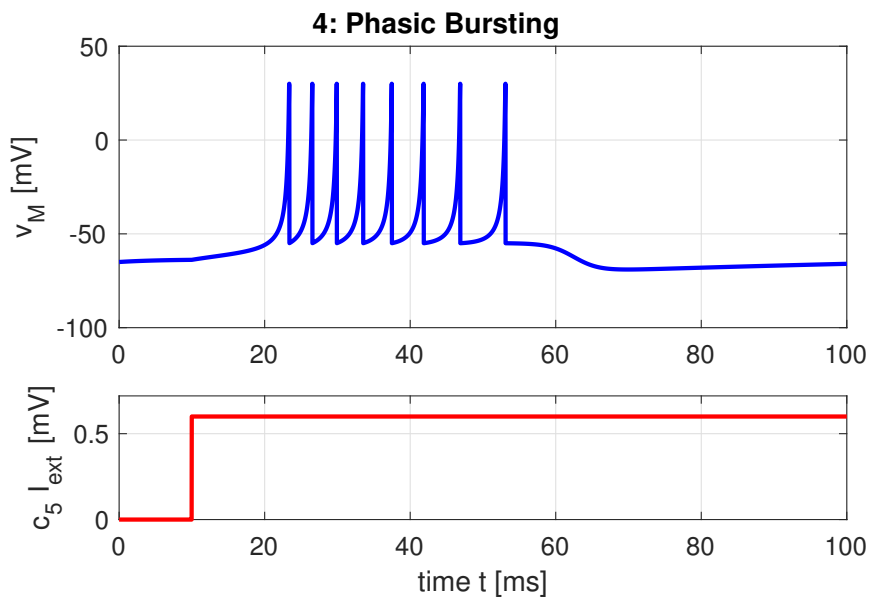


Phasic Bursting

$$a = 0.02 \quad b = 0.20 \quad c = -55 \quad d = 4$$

Phasic bursts are a set of closely spaced spikes that becomes quiescent with time. It is significant in neuroscience because it:

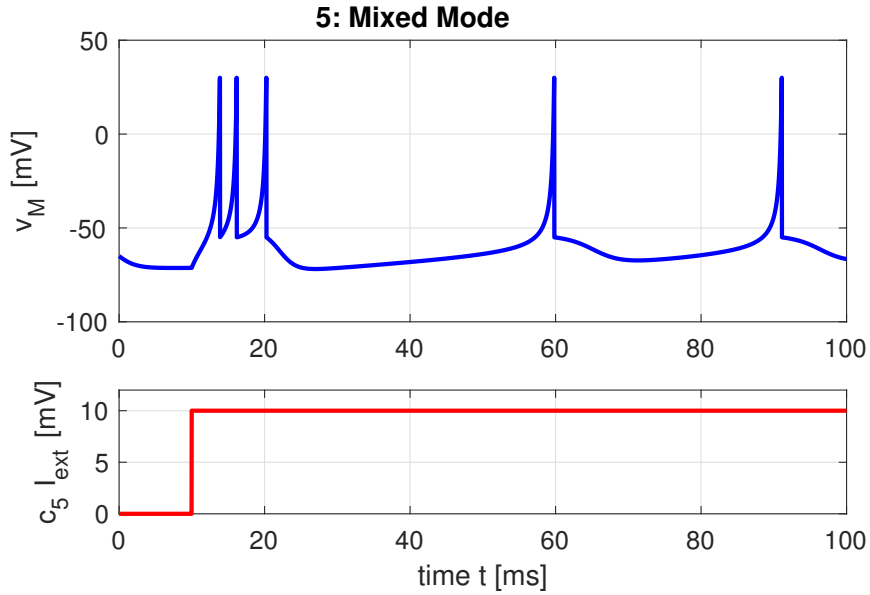
- may overcome synaptic failure and reduce noise
- post-synaptic signal is stronger than that of a single spike
- selective communication between neurons (inter-spike frequency within the burst encodes communication channels)



Mixed Model (Bursting Then Spiking)

$$a = 0.02 \quad b = 0.20 \quad c = -55 \quad d = 4$$

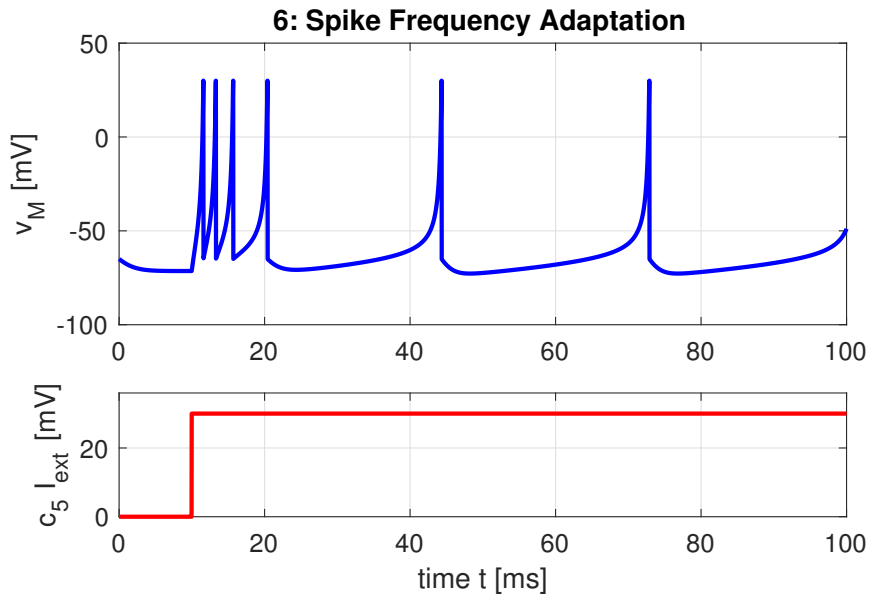
Excitatory neurons in the mammalian cortex can exhibit a phasic burst at the start of the stimulus and then switch to the tonic spiking mode.



Spike Frequency Adaptation

$$a = 0.01 \quad b = 0.20 \quad c = -65 \quad d = 8$$

The most common type of excitatory neuron in the mammalian neocortex, the regular spiking (RS) cell, fires tonic spikes with decreasing frequency. The frequency is relatively high at the onset of stimulation, and then it adapts. Low-threshold spiking (LTS) inhibitory neurons also have this property. The inter-spike frequency of such cells may encode the time elapsed since the onset of the input.



Network Model of Mammalian Cortex

The Izhikevich model for a neuron can be used to simulate a sparse network of 10^3 spiking cortical neurons with 10^6 synaptic connections. Based on the anatomy of the mammalian cortex, the ratio of excitatory to inhibitory neurons is taken to be 4 to 1 with inhibitory synaptic connections stronger than the excitatory synaptic connections. In the cortex, interneurons are responsible for inhibition comprise just a small fraction of the neurons, yet they have an important function in regulating activity of principal cells.

Regular spiking cells are used to model all excitatory neurons and fast spiking neurons for the inhibitory neurons. Each neuron receives a noisy thalamic input (pre-synaptic input into areas of the cerebral cortex from the thalamus). All neurons have different dynamics for heterogeneity. This is done by assigning to each neuron a range of values for the parameters a , b , c , and d using the uniformly distributed variables r_e and r_i which vary from 0 to 1.

Excitatory cells (variations in c and d)

- Regular spiking cells $a = 0.02$ $b = 0.20$ $c = -65$ $d = 8$
- Chattering cells $a = 0.02$ $b = 0.20$ $c = -50$ $d = 2$
- $c = -65 + 15r_e^2$
- $d = 8 - 6r_e^2$
 - $r_e = 0$ corresponds to regular spiking cell
 - $r_e = 1$ corresponds to chattering cell
 - r_e^2 used to bias the distribution toward regular spiking cells

Inhibitory cells (variations in a and b)

- Fast Spiking Cells
- $a = 0.02 + 0.08r_i$
- $b = 0.25 - 0.05r_i$
- $c = -65$
- $d = 2$

```
% E M Izhikevich: Network Model of Mammalian Cortex
```

```
close all
clc
clear
```

```
% INPUT PARAMETERS =====
Ne = 800; Ni = 200;           % number of excitatory & inhibitory neurons
Nt = 1000;                   % number of time steps
numE = 10; numI = Ne + 10;   % indices for one excitatory and one inhibitory
                             neuron
```

```
% Model parameters =====
re = rand(Ne,1); ri = rand(Ni,1); % random numbers
a = [0.02 * ones(Ne,1); 0.02 + 0.08*ri];
b = [0.20 *ones(Ne,1); 0.25 - 0.05*ri];
c = [-65+15*re.^2; -65*ones(Ni,1)];
d = [8-6*re.^2 ; 2*ones(Ni,1)];
S = [0.53*rand(Ne+Ni,Ne), -rand(Ne+Ni,Ni)]; % coupling strengths

v = -65*ones(Ne+Ni,1); % Initial values of v
u = b.*v;             % Initial values of u
firings = [];        % spike timings
```

```

vE = zeros(Nt,1); vI = zeros(Nt,1); % membrane potential of 2 neurons

% Time Evolution of Systems =====
for t = 1:Nt
    I = [5*randn(Ne,1);2*randn(Ni,1)]; % thalamic input
    fired = find(v>=30); % indices of spikes
    firings = [firings; t+0*fired,fired]; % time steps / fired neurons
    v(fired) = c(fired); % membrane potential
    u(fired) = u(fired)+d(fired); % recovery potential
    I = I+sum(S(:,fired),2); % thalamic + synaptic input

    v = v+0.5*(0.04*v.^2+5*v+140-u+I); % HALF-STEP: step 0.5 ms
    v = v+0.5*(0.04*v.^2+5*v+140-u+I); % for numerical stability
    u = u+a.*(b.*v-u);

    vE(t) = v(numE); % excitatory neuron
    vI(t) = v(numI); % inhibitory neuron
end

vE(vE > 30) = 30;
vI(vI > 30) = 30;

tS = 1:Nt; nF = zeros(Nt,1); % fired neurons at each time step
for t = 1 : Nt
    nF(t) = sum((firings(:,1) == t));
end

% GRAPHICS =====
figure(1) % raster plot of firing neurons / % firing neurons
set(gcf,'units','normalized','Position',[0.1 0.4 0.32,0.45]);
set(gca,'fontsize',12);
col = [1 0 0];
hP = subplot(2,1,1); % raster plot
plot(firings(:,1),firings(:,2),'.','color',col);
set(hP, 'Position',[0.1300 0.52 0.7750 0.4]);
ylabel('neuron #')

subplot(2,1,2); % percentage of firing neurons each time step
plot(tS,nF*100/(Ne+Ni),'color',col);
xlabel('time steps [ms]');
ylabel('% firing spikes')
grid on; box on;

figure(2) % membrane potential: excitatory neuron & inhibitory neuron
set(gcf,'units','normalized','Position',[0.3 0.1 0.32,0.30]);
set(gca,'fontsize',12);
plot(1:Nt,vE,'b');
hold on
plot(1:Nt,vI,'r');
xlabel('time steps [ms]');
ylabel('Membrane Voltage [mV]')
grid on; box on;
legend('excitatory','inhibitory');

```

A *pulse-coupled neural network (PCNN)* is a single layer, two-dimensional, laterally connected network of *integrate-and-fire* neurons. It is frequently used in image processing to detect segments or edges in an input image. The Izhikevich network model belongs to a class of pulse-coupled neural networks where the synaptic connection weights between the neurons are given by the matrix S so that firing of the n^{th} neuron instantaneously changes membrane potential variable v_m by the $S(m, n)$.



The model represents a network of 10^3 randomly coupled spiking neurons (800 excitatory and 200 inhibitory) with 10^6 synaptic connections. The top image shows a spike-train raster plot with episodes of *alpha rhythms* (single widely spaced peaks) and *gamma band rhythms* (double closely spaced peaks). The bottom image represents the typical spiking activity of an excitatory neuron and an inhibitory neuron (peaks normalized to +30 mV).

Neural activity in the neocortex is highly irregular and the origin of this irregular activity may be in the tight balance between excitatory and inhibitory synaptic inputs. Highly fluctuating net input currents whose means are below threshold results in action potentials being generated by the fluctuations. Neural activity in this state is *chaotic* in the sense that slight changes in initial conditions lead to drastically different patterns of spike times. We see that the excitatory neuron firing is about 7 Hz and the inhibitory firing rate is about 8 Hz. The network is *asynchronous* and neurons self-organize into assemblies in which different neurons emit spikes. Although the network is connected randomly and there is no synaptic plasticity, they exhibit collective rhythmic behavior in the frequency range corresponding to that of the mammalian cortex in the awake state.

The *alpha rhythm* corresponds to the normal bursts of electrical activity within the frequency range from 8 to 13 Hz in the cerebral cortex of a drowsy or inactive person. The *gamma rhythm* is the burst of electrical activity at higher frequencies (25 to 100 Hz, standard value is 40 Hz). The Izhikevich model may show some evidence of these rhythms.